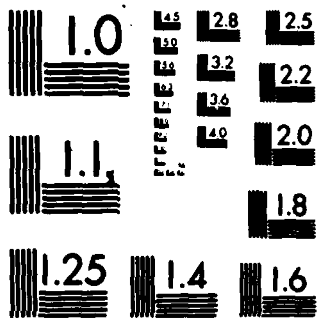

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THE HETEROSCEDASTIC METHOD: THEORY AND APPLICATIONS

Edward J. Dudewicz
Chairman, University Statistics Council
Department of Mathematics
Syracuse University

For the Period
October 1, 1979 - September 30, 1981

DEPARTMENT OF THE NAVY
Office of Naval Research
Arlington, Virginia 22217

Contract No. N00014-78-C-0543

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October 1, 1979 - September 30, 1981

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Research supported by Office of Naval Research
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INTRODUCTION

Statistical procedures for many problems have in the past usually, in fact almost exclusively, assumed that the random variables observed have equal variance-convariance matrices. In practice ~~we~~ the author often finds substantially unequal variance-covariance matrices arising in such practical problems as processing signals (e.g., sonar, radio, etc.) in noise, but classically only imprecise rule-of-thumb methods, or else methods involving some loss of information, were available for these problems (e.g., see Prokof'yev and Shishkin (1974)).

With the support of the Office of Naval Research, Contract No. N00014-78-C-0543, research was undertaken to develop the general theory for solving such problems and apply it to important problems. Areas studied and results obtained during the period October 1, 1979 to September 30, 1981 are summarized below. (Full results of completed studies have been communicated in technical reports to the ONR Statistics and Probability Program "Basic" and "Modeling and Estimation" distribution lists.) Lists of publications, reports and manuscripts arising from the project are then given.

Honors, awards, and other research activities of the Principal Investigator during this period are also summarized.

SUMMARY OF RESULTS

1. Heteroscedasticity in Quality Control Studies

Analysis of variance (ANOVA) is often used in quality control studies. It assumes equal variabilities within groups, and no exact procedures have been available for cases with unequal variabilities. Thus, even though experimenters are often cautioned that "the assumption of equal variability should be investigated" (e.g., p. 91 of Cochran and Cox (1957) or p. 46 of Section 27 of Juran, Gryna, and Bingham (1974)), no exact statistical procedures have been available for dealing with cases where one finds that variabilities are in fact unequal. In Bishop and Dudewicz (1979) we illustrate application of new exact procedures of Bishop and Dudewicz (1978) for ANOVA when treatment variabilities differ, using typical quality control situations, with explicit control of the power (as well as the level) of the test. Recommendations are given as to when one should abandon the common ANOVA procedures in favor of these new ones, with an indication of the costs one may incur by not doing so.

2. Multivariate Ranking and Selection

The problem of selecting that one of k multivariate normal populations which is "best" in some precise sense is an important one which arises frequently in practice (e.g. see Chapter 15, pp. 341-394, of Gibbons, Olkin, and Sobel (1977)). However, "The whole field is as yet undeveloped and the reader [of Gibbons, Olkin, and Sobel (1977), p. 390] is encouraged to regard this chapter [their Chapter 15] as an introduction to a wide area that will see considerable development in the future as more meaningful models are formulated." In Dudewicz and Taneja (1980) we have given this problem a new formulation which does not involve reducing the populations to univariate quantities. This formulation's solution is developed for known, and (using the Heteroscedastic Method) also for unknown, variance-covariance matrices. Preference reversals and arbitrary nonlinear preference functions are explictly allowed in this new theory.

While the nature of the preference function is not a chief point of interest in this paper, this work holds for any and every such function. The specification and/or elicitation of such functions is part of the field of decision theory. Up to now it has dealt with only relatively "simple" types of functions. (For some illustrative papers developing nonadditive (but still relatively simple) utility theory, see Farquhar (1976) and Fishburn (1977).

For some typical applications of this utility theory in practical situations, see Giauque and Peebles (1976) and Krischer (1976). For some consideration of assessment of multiattribute cardinal utility functions see Kirkwood (1976). For research papers and a survey of the field of multiple criteria/objective decision making, see respectively Starr and Zeleny (1977) and Hwang and Masud (1979).) One could investigate such specifications as (e.g.) polynomial functions, but no one has yet been able to do so. The present paper investigates how one can do statistical inference with any such function (even if not completely elicited or known), and this has been hitherto unavailable.

3. Estimation of the Larger Mean

The ranking and selection problem is well-known (see Bechhofer, Kiefer, and Sobel (1968)), and has been the topic of over 1000 papers since 1950. Once the decision has been made as to which of the populations has the largest mean, the question arises, "How large is this largest mean?" (For two examples of application, in hearing testing and weakest-link theory, see Blumenthal and Cohen (1968).)

In Dhariyal, Dudewicz, and Blumenthal (1980) we study this problem and enlarge the class of estimators of the larger mean.

We investigate the behavior of a class of estimators called Maximum Probability Estimators (MPE's) introduced by Weiss and Wolfowitz (1967), which was first studied for ranked means by Dudewicz (1973). Then a new class of estimators called Iterated Bias Elimination Estimators (IBEE's) is introduced and investigated, including comparisons with MPE's.

4. Complete Categorized Guide to Ranking and Selection

While books on ranking and selection (Bechhofer, Kiefer, and Sobel (1968); Gupta and Panchapakesan (1979)) have become available in recent years, and have had reasonable bibliographies included in them, even that of Gupta and Panchapakesan (1979) appears to contain only about 60% of the relevant papers, which number over 1000. Also, these are nowhere available in a categorized manner which would make the related research in any subcategory easily available to a practitioner or theoretician needing it. Culminating an effort begun with the issuance of "A Categorized Bibliography on Multiple-Decision (Ranking and Selection) Procedures" by E. J. Dudewicz in November 1968, major new effort has expanded this to an up-to-date research tool, reviewed by over 75 of the leaders of the field in a preliminary

version, including 46 categories and indices. This monograph, Dudewicz and Koo (1982), of over 600 pages, appeared in 1982, in time for the First International IPASRAS (Inference Procedures Associated with Statistical Ranking and Selection) Advanced Seminar/Workshop, held July 1982 at the East-West Center, Honolulu, where it was acclaimed by such leaders of the field as Dr. Milton Sobel.

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One copy of the Dudewicz and Koo (1982) book, and several copies of others of the items (those which were not submitted with the 1980 Progress Report on this contract) are appended to this report. Other reprints and preprints are available on request.

HONORS, AWARDS, AND OTHER RESEARCH ACTIVITIES

Edward J. Dudewicz has given colloquium and seminar talks at Katholieke Universiteit Leuven (Department of Mathematics), Western Illinois University (Department of Mathematics), Rutgers University (Department of Statistics), Denison University (Department of Mathematical Sciences) and University of Texas at Arlington (Department of Mathematics) during this period, in many cases detailing the new theory of multivariate ranking and selection.

He has also given an Invited Lecture to the Central Illinois Chapter of the American Statistical Association; been Moderator of a Short Course on "How to Perform Continuous Sampling" at the 1980 Annual Technical Conference of the American Society for Quality Control in Atlanta, Georgia, at which he also was Moderator of a Tutorial on "How to Test Normality and Other Distributional Assumptions" by Samuel S. Shapiro, and where he also spoke.

He was Organizer and Speaker, Invited Paper Session on Design and Analysis of Heteroscedastic Data, American Statistical Association Annual Meeting, Houston, Texas (August 1980), in which other participants were R. E. Bechhofer, S. R. Dalal, T. J. Santner, and A. C. Tamhane.

He was Invited Speaker, as a guest of the Statistical Society of Australia, at the 5th Australian Statistical Conference, University of New South Wales, Sydney, Australia (August 1980), and spoke at Australian universities while on this trip, including the University of Queensland, and the University of New England.

He was accorded a 1978 Shewell Award competition Certificate of Recognition for the paper "Design and analysis of heteroscedastic experiments." Presentation was made at the 1979 Fall Technical Conference of CD of ASQC and SPES of ASA, held in Minneapolis, Minnesota. He has been Editor of the American Journal of Mathematical and Management Sciences since 1981, and also serves as an Editor of Statistics & Decisions.

Both E. C. van der Meulen and H. J. Chen participated in the project research activities during visits to Ohio State as Visiting Professors. These activities have led to research reports, as have those of Graduate Research Assistants M. G. Sri Ram and B. K. Taneja.

The accomplishments of the Principal Investigator, Dr. Edward J. Dudewicz, were recognized by election as a Fellow of the Institute of Mathematical Statistics in 1981. He was also elected a Fellow of the American Statistical Association in 1981.